

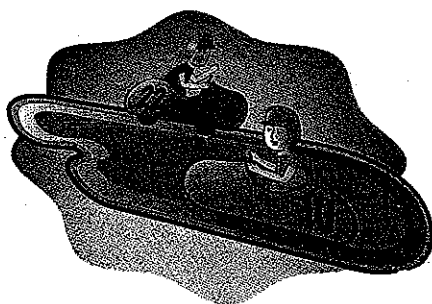


The relationship between the area of a sector and the central angle of its arc is nicer than the relationship between the area of a segment and the central angle of its arc. What does nicer mean in this case? (Hint: Think about ratios of measures.)

Here's a story to illustrate how knowing about arcs and sectors can be useful.

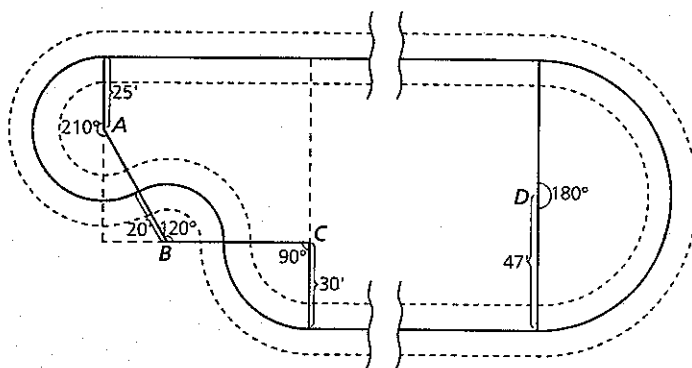
A Go-Kart Track (Part I)

Nip and Tuck, the Racer twins, have decided to build a Go-Kart track. They have found a simple design in their favorite Go-Kart magazine. Its diagram is shown in Display 4.37.



Notes with the diagram explain that the curves are circular arcs, to make the track easy to build. The dark line is the center of the track. The dashed lines are the edges, 10 feet away on each side of the center, all the way around. The notes go on to describe how to bank the turns, how to ease the curvature slightly when connecting the circular parts to the straightaways, etc., but these things will not concern us here.

The breaks in the straightaways, shown by the wavy lines, indicate that the length of the track may be adjusted, depending on how long these straight parts are made.



Go-Kart track

As a first step in building their Go-Kart track, Nip and Tuck decide they need a larger scale drawing of the center line of Display 4.37. Make one for them. Draw it on a piece of $\frac{1}{4}$ inch ruled graph paper, to the scale of $\frac{1}{4}$ inch = 5 feet.



(Hint: If you turn your $8\frac{1}{2} \times 11$ inch piece of graph paper sideways and start by placing point A about 3 inches down from the top left corner and 2 inches to the right, you should have just enough room.)

Nip and Tuck like this track design because its length can be adjusted. They want to build a track exactly $\frac{1}{10}$ of a mile long. This will allow them to figure out easily each Go-Kart's average speed in miles per hour. Do you see how?

How long should they make the straightaways?

Nip and Tuck's question of how long to make the straightaways is difficult to answer all at once. Break it into smaller questions that are easier to handle. Then answer at least one of them.

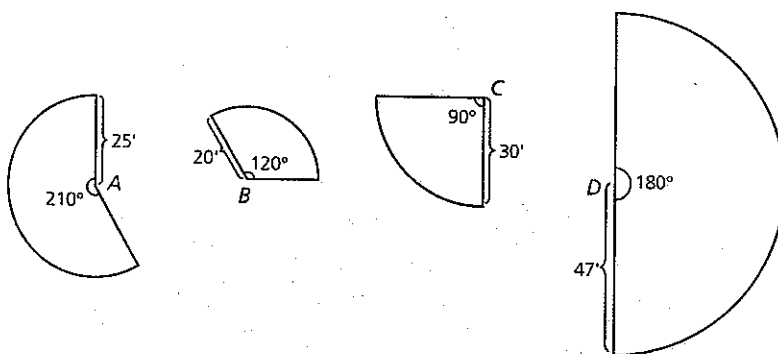


Before deciding how long to make Nip and Tuck's straightaways, we have to know the lengths of the four circular arcs in the track. They are shown separately in Display 4.38. Let's take them one at a time.

Thinking Tip

Ask simpler questions.

When faced with a question you can't answer, keep asking other questions about it until you find one you can answer. Then work from there.



The circular arcs

Display 4.38



Copy the table in Display 4.39. As you work through these questions, keep track of your answers by filling in your copy of Display 4.39. Use your calculator to help with the arithmetic, and round your answers to one decimal place.

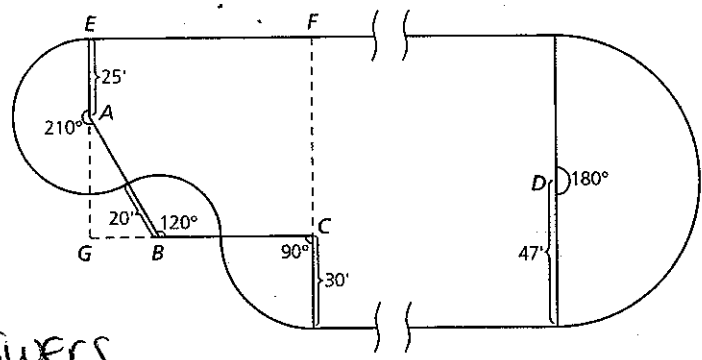
1. Which of the four arc lengths of Display 4.38 do you think is easiest to find? Why?
2. Can you think of a way to find any of these lengths? If you can, do it, and explain your method.
3. If you completed the circle centered at D , what would its circumference be?
4. What is the length of the 180° arc centered at D ? Explain.
5. If you completed the circle centered at C , what would its circumference be?
6. What is the length of the 90° arc centered at C ? Explain.
7. If you completed the circle centered at B , what would its circumference be?
8. What is the length of the 120° arc centered at B ? Explain.
9. Use your thinking from the previous questions to describe a process for finding the length of any arc from its radius and central angle. Explain your process in words and then in a formula. Use your process to find the length of the 210° arc centered at A .

Center	Radius (r)	Circumference	Central Angle	Arc Length
D	47		180°	
C				
B				
A				
\circ	r		x°	

X

Now your drawing should look like Display 4.40.

1. Why are the lengths of EF and GC equal?
answer: It is a rectangle
2. How long is BC ? Why?
3. How long is AB ? Why?
4. How long is GB ? Why? (It is not 20)
5. How long is GC ? Why?
6. How long is EF ? Why?



Display 4.40

Check:
Part I has 15 answers
+

for a total
of 16 answers

Now we have all the pieces needed to answer Nip and Tuck's question. Here it is again:

Nip and Tuck want to make their Go-Kart track exactly $\frac{1}{10}$ of a mile long. How long should they make the straightaways?

A Go-Kart Track (Part II)

With the track design settled, Nip and Tuck are planning the cost of its construction. They want to cover the infield (the region inside the track) with sod (grass). The sod is sold by the square foot and is expensive, so they need an accurate measure of the area of the infield.

They begin by drawing the inner border of the track, 10 feet away from the center line all around. Then they divide the infield into 7 pieces and write

